On Errors arising in the Measurement of Unsymmetrical Spectrum Lines.

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It is generally recognised that one of the most serious sources of error in relative determinations of the wave-lengths of spectrum lines by micrometric measurements of the positions of the lines on photographic plates lies in the mechanical and other displacements, which result in errors in the superposition or juxtaposition of the spectrum under investigation and the standard spectrum, the wave-lengths of which are supposed to be known. Thus, if a spectrum to be measured is photographed on a plate and an iron arc spectrum is then photographed in juxtaposition, an error may arise, either from some flexure or mechanical displacement between the two exposures or from a gradual change of temperature of the dispersing prism or grating, resulting in a continuous drift of the spectrum across the plate. Even when the utmost care is taken to avoid these displacements by rigid design of the spectrograph and proper control of temperature, it is hardly ever possible to eliminate them entirely, and it is necessary to follow some procedure which will reduce the errors arising from them to a minimum.

A method which is commonly used is to expose the comparison spectrum in two parts. For example, a comparison spectrum with one-half the total exposure required may be taken first; the spectrum to be measured is then exposed for an appropriate time, and finally the second half-exposure of the comparison spectrum is superposed on the first. Images of the comparison lines (and in the case of a long exposure of the lines to be measured) would be somewhat broadened, but it might at first seem that the maxima of the lines in the two spectra would be in correct juxtaposition, supposing that there has been a constant drift of the spectrum across the plate owing to a gradual change of temperature. St. John and Babcock,* in a study of the pole effect in the iron arc, used a system of reflecting prisms to project on the slit simultaneously the light from the neighbourhood of the pole in juxtaposition with that from the centre of the arc, the intensities being equalised by means of a rotating sector, which occulted for an appropriate period the more intense light from the neighbourhood of the pole.

It might at first appear that in the case in which a source emitted both standard lines and the lines under investigation the errors due to displacements would be entirely innocuous. It is unfortunate that this is true only for the special case in which the lines of both spectra are symmetrical, and it is by no means certain that this condition is ever fulfilled, excepting in the case of the spectra of gases contained in vacuum tubes at low pressures and excited by uncondensed discharges, where the sole cause of broadening is known to be that due to the motions of the radiating particles in the line of sight operating in accordance with Doppler's principle. Although it is obvious that errors must arise in the case in which the lines are not symmetrical, their importance does not appear to have been recognised fully.

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* 'Astrophys. Journ.,' vol. 42, p. 231 (1915).

As an illustration of the extreme case we may consider (fig. 1) two symmetrical lines with maxima at a, a, and between them a completely unsymmetrical line with maximum at b, the ordinates representing the intensity and the abscissæ the wave-length scale. If during an exposure the lines drift across the photographic plate with uniform velocity, so that at the end of the exposure their true maxima lie at a_1, a_1, and b_1, it is clear that on the photographic plate the maxima will appear to be at a_2, a_2, and b_1, and if a, a, are used as standards for determining the wave-length of b, the wave-length so found will be in error by as much as one-half of the total shift. It is assumed that in the measurement of spectrograms the cross-wires of the microscope are set on the blackest part of the line, but a discussion of the extent to which this is reached in practice is beyond the scope of the present note.

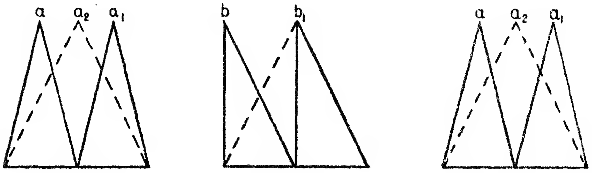

Fig. 1.

The above example is given merely to illustrate the principles on which the errors arise, and although the displacements which occur in practice must be of so varied a character that it is not possible to discuss them in detail, it is of interest to consider the case of a uniform displacement, and it will be shown that the errors can be expressed in a general form which is independent of the intensity distribution. In a previous investigation* the limitation which the resolving power of the spectroscope imposes on the accuracy of wave-length determinations of unsymmetrical lines has been considered, and it was found convenient to represent the intensity distribution of lines by the law $I = e^{-kx^2}$, the asymmetry being reproduced by adopting different values of k on the two sides of the maximum. It is convenient to define as the "index of symmetry," I, the ratio of the "half-widths" x_1, x_2, on either side of the maximum, these being the values of x which make I equal to $\frac{1}{2}$, the value of I at the maximum being taken as unity.

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* Nicholson and Merton, 'Roy. Soc. Proc.,' A, vol. 98, p. 261 (1920).

In fig. 2 the unsymmetrical line having a maximum at O is supposed to move with uniform velocity during an exposure, until at the end of the exposure the maximum lies at A. The position of the maximum on the plate lies at the intersection of the two intensity-distribution curves at the beginning and end of the exposure. It can easily be shown that this result is perfectly general if the curves intersect, and is independent of their shape. The intensity at X is proportional to the area bounded by OBX_1B_1A and is smaller at any other point. Thus at P the intensity is proportional to the areas $OBQP + PP_1B_1A$, and is evidently less than the intensity at X by an amount proportional to the area X_1P_1Q. In the particular case in which the two values of X for each intensity are in a constant ratio, *e.g.*, if the curve is of the exponential form given above, the position of the maximum is given by $OX/XA = x_1/x_2$, and $OX = OA/(1 + x_2/x_1) = OA/(1 + 1/I)$.

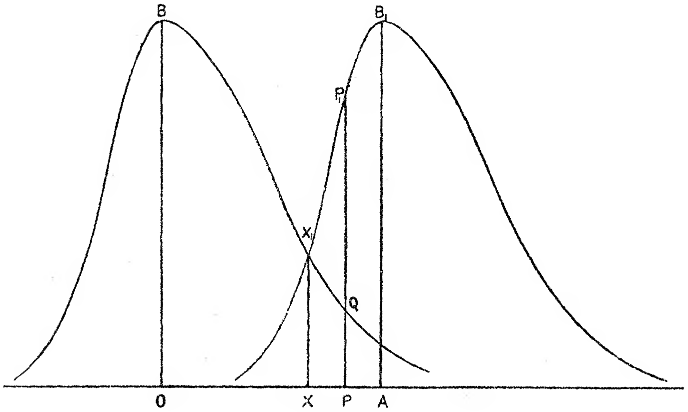

Fig. 2.

From this result it follows, for example, that if the index of asymmetry of such a line were equal to 5, an error of one-third of the displacement would result in the determination of its wave-length by comparison with two symmetrical lines on either side of it.

One of the main causes of the broadening of spectrum lines is that due to the electrical resolution of the lines, caused by the electric field of neighbouring charged particles on the radiating atoms, and since in a number of cases the effect of the electric field is to produce a simple displacement to one side, or a resolution into components on one side only, the index of asymmetry for such lines might be very great. In the case of a broadening due to this cause, the constancy of the ratio of the two values of x for any intensity follows from the proportionality which is usually found between the intensity of the electric field and the resolution of the lines.

St. John and Babcock (*loc. cit.*) in their investigation of the pole effect have made use of the ingenious device of inserting a bulb containing iodine at a low pressure between the iron arc and the spectrograph, and a number of lines were found to have one or more very narrow iodine absorption lines superposed on them. This procedure, however, does not automatically eliminate the errors arising from instrumental displacements, and it can be shown that if the emission line is unsymmetrical and a displacement occurs during the course of the exposure, the minimum of the absorption line, actually lying on one side of the emission maximum, may appear on the plate on the other side of the maximum.

It seems that instrumental displacements can be rendered innocuous only in the case of spectra whose lines are known to be symmetrical, such as the spectra of gases at low pressures referred to above, and such spectra are, therefore, to be preferred as standards of wave-lengths, rather than arc spectra at atmospheric pressure, where it is known that many of the lines are unsymmetrical. The asymmetry of the lines in arc spectra may depend on the density of the vapour in the arc, but in measurements of such lines displacements will have the effect that all the errors will be in the direction in which the lines show asymmetry. This may indeed be contributory to the fact that there are series of lines which cannot be represented satisfactorily by any of the formulæ which are generally used to connect the individual members of spectrum series, for Royds* has shown that in some series the earlier members may show asymmetry in one direction whilst in the later members the direction of the asymmetry is reversed.

These considerations seem to be relevant in all cases where the detection and measurement of minute changes of wave-length are involved. It is to be regretted that at the present time we have very little precise information as to the distribution of intensity in broadened spectrum lines, but it is possible that micrometric measurements may be carried sometimes to a higher degree of precision that the rigidity and efficiency of temperature control of the spectrograph can justify.

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* 'Astrophys. Journ.,' vol. 41, p. 154 (1915).